Quiz #6 Math 124 Autumn 2016

Name: Student Number:

No books or notes allowed on this exam.

Find the absolute maximum and absolute minimum of

$$f(x) = x + 2\cot^{-1} x$$

on the interval $[0, 4]$. Use sentences to justify your answer (don't just circle a number, but use the reasoning we learned in class.)

Solution : [J. Stewart, Page 278] **The Closed Interval Method** To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
   Set $f'(x) = 0$.
   Solve for the critical numbers:

   $$\begin{aligned} f'(x) = 1 + 2 \cdot \left(-\frac{1}{1+x^2}\right) &= 0 \\ 1 &= \frac{2}{1+x^2} \\ 1 + x^2 &= 2 \\ x^2 &= 1 \Longrightarrow x = \pm 1. \end{aligned}$$

   Since -1 is not in the domain, the interval $[0, 4]$, we have $x = 1$ as our critical number. Plug in $x = 1$ in f. We have

   $$f(1) = 1 + 2\cot^{-1} 1 = 1 + 2 \cdot \frac{\pi}{4} = 1 + \frac{\pi}{2}.$$

2. (3 points) Find the values of f at the endpoints of the interval.

   $$\begin{aligned} f(0) &= 0 + 2\cot^{-1} 0 = \pi. \\ f(4) &= 4 + 2\cot^{-1} 4. \qquad \text{(No need to simplify further.)} \end{aligned}$$

3. (1 point) The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.
   To get the credit in this part, you only need to state the step in sentences. However, there is a way to find the largest value and the smallest value without using the calculator. Note that

   $$1 + \frac{\pi}{2} < \pi < 4 + 2\cot^{-1} 4.$$

   The first inequality is true because $1 < \frac{\pi}{2}$. The second inequality is true because $\pi < 4$ and $0 < \cot^{-1} 4$.
   Therefore, $f(1)$ is the absolute minimum and $f(4)$ is the absolute maximum.